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A quarter-car suspension system: car body mass estimator and sliding mode control

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Abstract

The purpose of this paper is to present a robust control scheme for a quarter-car suspension system under a road disturbance profile. Here a linear mathematical model is presented in order to design a sliding mode controller that allows avoid the induced road variations over the car body. Novelty of this paper is given by the algebraic estimator used to find the car body mass of the quarter-car system, the results show that the main control objective can be reached: the passengers comfort.

Keywords: sliding mode control, active suspension, simulation

1. Introduction

An ideal suspension car system should be able to isolate the car body from the perturbations induced by the road. In general, the suspension systems can be classified, base on the external power input, as passive, semi-active and active. A passive suspension system is a conventional suspension used in almost the commercial vehicles and motion of car body is variable subject to road conditions. The semi-active suspension system has the same elements of conventional system, but the damper has two or more selectable damping rates and requires a high force at low velocities and a low force at high velocities, and be able to move rapidly between the two. An active suspension has an actuator that allows improve the passenger comfort due this element is

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placed in parallel with the damper and the spring between the car body (sprung mass) and the wheel (unsprung mass).

Typically, active suspension systems include actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control strategies such as adaptive control presented by Nugroho et al. [1], fuzzy control in Ranjbar-Sabrine et al. [2] and optimal control developed by Paschedag et al. [3] have been proposed in the past years to control the active suspension system.

In this paper the robust control design proposed, based on the sliding mode control technique by Utkin [4], allows the suppression of the road perturbations over the body of a car, increasing the passengers comfort. Also, based on the algebraic approach proposed by Fliess et al. [5,6], a mass sprung estimator is presented.

2. System dynamics

A quarter-car suspension system shown in Fig.1 is used to simulate the control system. The dynamics equations of the suspension system using Newton or Euler-Lagrange methodology, as presented by Fateh and Alavi in [7], are of the following form

$$m_s\ddot{z}_s = -b_s(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) + f_a \tag{1}$$

$$m_u\ddot{z}_u = b_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) - f_a + b_t(\dot{z}_r - \dot{z}_u) + k_t(z_r - z_u) \tag{2}$$

were m_s, m_u, k_s, k_t, b_s and b_t denote the mass, stiffness and the damping rate of the sprung and unsprung elements, respectively. The road variations are represented by z_r and the variables z_s and z_u are the body and wheel displacements, respectively. The system is equipped with an active damper placed between the sprung and unsprung masses to exert the required control force f_a. Some of the road variations are introduced using the undamped natural frequency of the system.

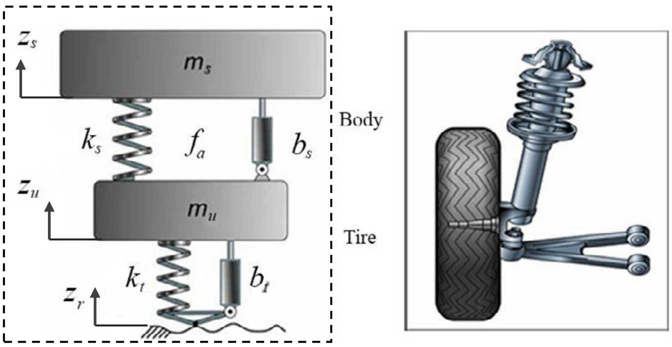


Fig. 1 A quarter-car model of suspension system

In order to obtain the undamped natural frequencies of the unperturbed system the parameters b_s, b_t, f_a and z_r are equal to zero in (1) and (2), obtaining the following homogeneous equations

$$m_s\ddot{z}_s + k_s(z_s - z_u) = 0 \tag{3}$$

$$m_u\ddot{z}_u + k_s(z_u - z_s) + k_t z_u = 0 \tag{4}$$

Proposing the solutions $z_s = z_s e^{j\omega t}$ and $z_u = z_u e^{j\omega t}$ (3) and (4) can be written in the following form

$$[A][z]=\begin{bmatrix}-m_s\omega^2+k_s & -k_s\\ k_s & -m_u\omega^2+k_s+k_u\end{bmatrix}\begin{bmatrix}z_s\\ z_u\end{bmatrix} \tag{5}$$

where *A* is the called stiffness matrix of the system. The determinant of matrix A is given by

$$\det(A)=m_s m_u\omega^4-m_s(k_s+k_u+m_u k_s)\omega^2+k_s k_u \tag{6}$$

Equating (6) to zero and solving for ω the two undamped natural frequencies are obtained.

3. Sliding mode control design

The main aim of control design is to provide the desired dynamic behavior of vehicle under road variations. According to Chavez et al. in [8], the sliding modes technique allows fulfill the control objective if the next sliding surface is used

$$\sigma=(\dot{z}_s-\dot{z}_d)+c_1(z_s-z_d)+c_0\int(z_s-z_d) \tag{7}$$

where z_d represents the desired vehicle behavior, c_1 and c_2 are positive constants to be determined. Differentiating the equation (7) once

$$\dot{\sigma}=(\ddot{z}_s-\ddot{z}_d)+c_1(\dot{z}_s-\dot{z}_d)+c_0(z_s-z_d) \tag{8}$$

and replacing from (1) the second derivative of the sprung mass displacement, the dynamics of the sliding surface is given by

$$\dot{\sigma}=\frac{1}{m_s}\left[-b_s(\dot{z}_s-\dot{z}_u)-k_s(z_s-z_u)+f_a\right]-\ddot{z}_d+c_1(\dot{z}_s-\dot{z}_d)+c_0(z_s-z_d) \tag{9}$$

When $\dot{\sigma}=0$, the so called equivalent control can be obtained as

$$(f_a)_{eq}=k_s(z_s-z_u)+b_s(\dot{z}_s-\dot{z}_u)+m_s(\ddot{z}_d-c_1(\dot{z}_s-\dot{z}_d)-c_0(z_s-z_d)) \tag{10}$$

which restricts the system dynamics when the sliding surface has been reached. To force the system dynamics to reach the sliding surface the following attractive control is used

$$(f_a)_n=-m_s\mathrm{Lsign}(\sigma) \tag{11}$$

where L is a positive constant. Finally, the sliding mode controller, given by the sum of the equivalent and attractive control is the following

$$f_a=k_s(z_s-z_u)+b_s(\dot{z}_s-\dot{z}_u)+m_s(\ddot{z}_d-c_1(\dot{z}_s-\dot{z}_d)-c_0(z_s-z_d))-m_s\mathrm{Lsign}(\sigma) \tag{12}$$

4. Sprung mass estimator

The basic idea of the algebraic estimator is based on the identification method proposed and analyzed by Fliess et al. [5, 6 ,9]. In order to obtain the sprung mass estimator, the differential equation (1) is described in notation of operational calculus as follows

$$m_s\left[s^2 Z_s(s) - s z_s(0) - \dot{z}_s(0)\right] + b_s\left[sZ_s(s) - z_s(0) - sZ_u(s) + z_u(0)\right] + k_s\left[Z_s(s) - Z_u(s)\right] = F_a(s) \tag{13}$$

where $z_s(0),\ \dot{z}_s(0), z_u(0), \dot{z}_u(0)$ are the system initial conditions. In order to eliminate the dependence of unknown constants, the equation (13) is differentiated twice with respect to the variable s, resulting in

$$m_s\left(2Z_s + 4s\frac{dZ_s}{ds} + s^2\frac{d^2 Z}{ds^2}\right) + b_s\left(2\frac{dZ_s}{ds} + s\frac{d^2 Z}{ds^2} - 2\frac{dZ_u}{ds} - s\frac{d^2 U}{ds^2}\right) + k_s\left(\frac{d^2 Z}{ds^2} - \frac{d^2 U}{ds^2}\right) = \frac{d^2 F_a}{ds^2} \tag{14}$$

Now, multiplying (14) by s^{-2} one obtains that

$$\begin{aligned} & m_s\left(2s^{-2}Z_s + 4s^{-1}\frac{dZ_s}{ds} + \frac{d^2 Z}{ds^2}\right) + b_s\left(2s^{-2}\frac{dZ_s}{ds} + s^{-1}\frac{d^2 Z}{ds^2} - 2s^{-2}\frac{dZ_u}{ds} - s^{-1}\frac{d^2 U}{ds^2}\right) \\ & + k_s\left(s^{-2}\frac{d^2 Z}{ds^2} - s^{-2}\frac{d^2 U}{ds^2}\right) = s^{-2}\frac{d^2 F_a}{ds^2} \end{aligned} \tag{15}$$

and transforming back to the time domain leads to the integral equation

$$m_s\left(2\iint z_s + 4\int t z_s + t^2 z_s\right) + b_s\left(2\iint t z_s + \int t^2 z_s - 2\iint t z_u - \int t^2 z_u\right) + k_s\left(\iint t^2 z_s - \iint t^2 z_u\right) = \iint t^2 f_a \tag{16}$$

By solving (16) it is obtained the following estimator for the unknown sprung mass m_s

$$m_s = \frac{\iint t^2 f_a - b_s\left(2\iint t z_s + \int t^2 z_s - 2\iint t z_u - \int t^2 z_u\right) - k_s\left(\iint t^2 z_s - \iint t^2 z_u\right)}{2\iint z_s + 4\int t z_s + t^2 z_s} \tag{20}$$

It is evident that the knowledge of the sprung and unsprung masses displacements and the control force is required.

5. Simulation results

The simulation results were obtained by means of MATLAB/Symulink®, with the Runge-Kutta numerical method and fixed integration step of 1 ms. The numerical values for the the quarter-car suspension, presented by Arbelaez et al. [10], and the control parameters are shown in table 1.

Table 1.Parametes: Quarter-car suspension system and control

Parameter	Value	Units
Sprung mass (m_s)	208	kg
Unsprung mass (m_u)	28	kg
Spring stiffness (k_s)	18,709	N/m
Damping constant (b_s)	1,300	N·s/m
Tire stiffness (k_t)	127,200	N/m
Tire damping (b_t)	10	N·s/m
Control parameters (c_0,c_1,L)	100,30,0.1	

The road perturbations profile is shown in Fig. 2. One can notice the three different amplitudes and frequencies acting over the quarter car model. The first two signals represent a bumpy road and the third signal between 6 and 12 seconds represents a speed reducer in the road.

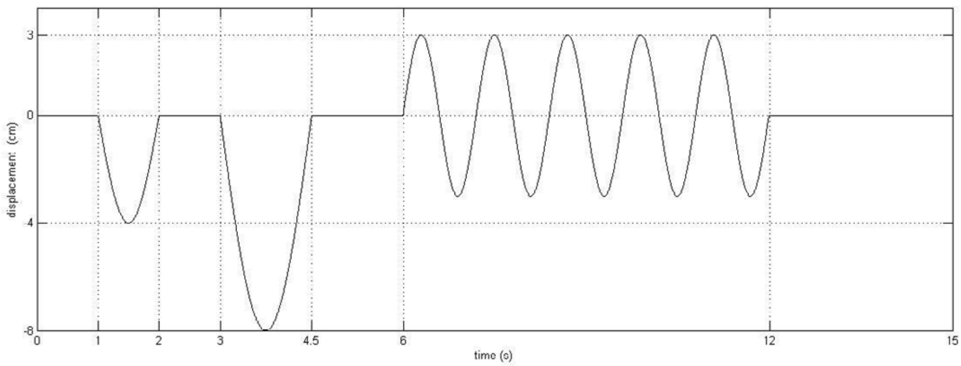


Fig. 2. Road perturbations

In this simulation, the desired position for the sprung mass is a constant value of 5 cm. The free displacement versus the controlled displacement for the sprung mass is shown in Fig.3.

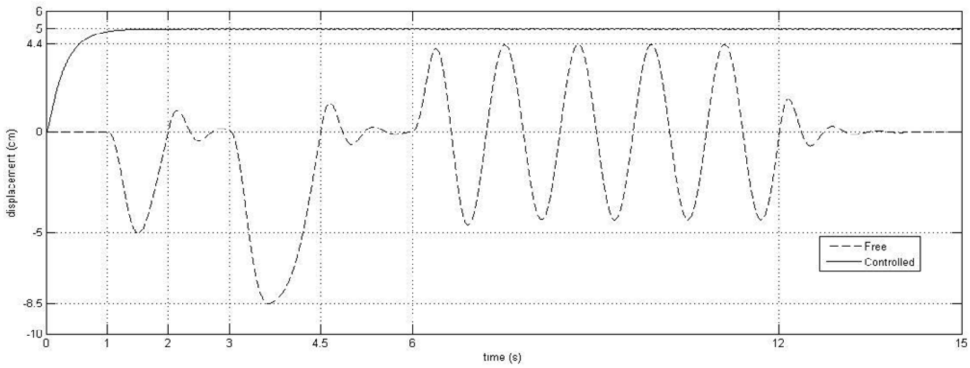


Fig. 3. Displacement of sprung mass: free vs controlled

In Fig. 4 one can notice that the unsprung mass displacement oscillates even in the controlled system, this is because the sliding surface only requires that the sprung mass displacement reaches the desired value.

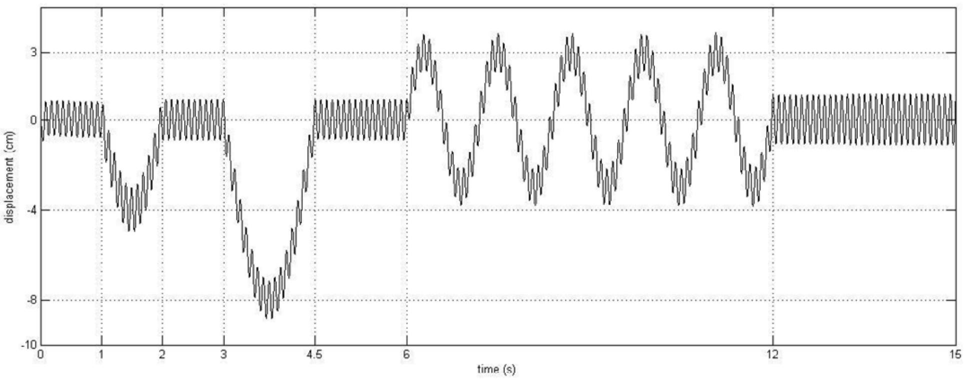
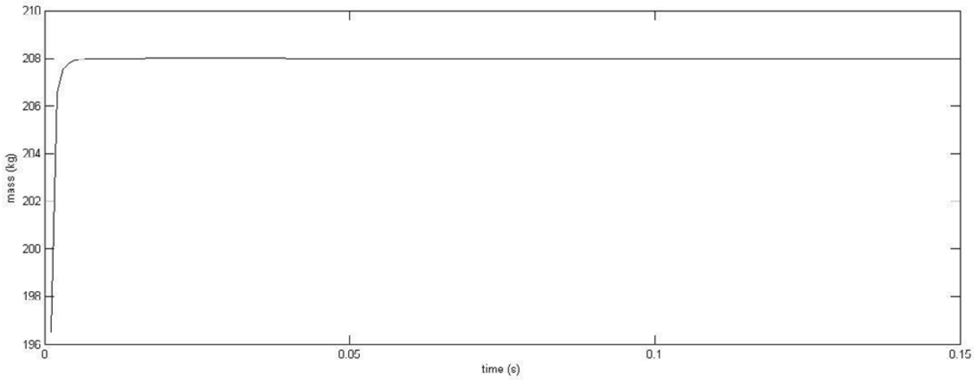


Fig. 4. Displacement of unsprung mass

The sprung mass estimator behavior is shown in Fig. 5. One can notice that the estimator reaches the sprung mass value of 208 kg in a small time of about 0.01 seconds, which allows using it in a new robust control scheme with parameter identification.



Conclusions and further work

The paper presents a control option for an active suspension system. The proposed sliding mode controller is robust under road variations and the simulation results prove that algebraic estimator is an available election for use into controller designs.

The further work is directly related with the algebraic estimation of spring stiffness and damping values. Once having the parameters estimation, the control scheme could be implemented it in a real quarter-car suspension system.

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